Study of Modulus of Elasticity of RC Beam under Flexural Loading using ANSYS

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Abstract : With the recent advancement in the structural design of concrete structure it has become very essential to know the elastic properties of reinforced concrete rather than using elastic property of plain concrete in which only cross area of plain concrete is taken into consideration whereas the effect of reinforcement bars and tie bars which confines concrete are not considered. As per the current IS: 456-2000 codal provisions the modulus of elasticity of concrete is expressed as a function of grade of concrete. The aim of this study is to determine Modulus of Elasticity of Reinforced Concrete by non-linear Finite Element Analysis (model size 250 x 300 x 4800 mm) with different grades of concrete (M20, M25, M30) and percentage of tension reinforcement varying from 0.54 to 1.26% using ANSYS.

Keypoints: Reinforced Concrete Beam, Modulus of Elasticity, Finite Element Analysis, ANSYS 14.0.

1. Introduction

Reinforced concrete is one of the most important building materials and is widely used in many types of engineering structures. The economy, the efficiency, the strength and the stiffness of reinforced concrete make it an attractive material for a wide range of structural applications. The ultimate objective of the designer is to create a structure that is safe and economical. The safety and serviceability assessment of the structures necessitate the development of accurate and reliable methods and models for their analysis. The rise in cost of materials used in structures and labour costs encourage engineers to seek more economical alternative designs often resorting to innovative construction methods but without lowering the safety of the structure. In addition, the extent and impact of disaster in terms of human and economical loss in the event of structural failure promote designers to check the design thoroughly. The development of numerical models for the analysis of the response of RC structures is complicated due to following reasons.

a) Reinforced concrete is a composite material made up of concrete and steel, two materials with very different physical and mechanical behavior;
b) Concrete exhibits nonlinear behavior even under low level loading due to nonlinear material behavior, environmental effects, cracking, biaxial stiffening and strain softening;
c) Reinforcing steel and concrete interact in a complex way through bond-slip and aggregate interlock.

These complex phenomena have led engineers in the past to rely heavily on empirical formulas for the design of concrete structures, which were derived from numerous experiments. With the advent of digital computers and powerful methods of analysis, much effort to develop analytical solutions which would obviate the need for experiments have been under taken by investigators.

2. Methodology

For the purpose of study the following specification is made for doubly reinforced section.

(A) Beam size – the width and depth of the beam is 250mm and 300mm respectively.
(B) Span length – 4800mm
(C) Grade of concrete – M 20 for group A.
M 25 for group B.
M 30 for group C.
(D) Grade of steel – Fe 415 for all group specimen
(E) Percentage of steel– 0.54, 0.69,0.80, 0.84,1.11 and 1.26 % are taken.
(F) Shear reinforcement – 2 legged 8mm dia. bar
(G) Spacing of stirrups – 200 mm c/c.

2.1 Finite Element Modelling of Reinforced Concrete Beam

Finite Element Analysis (FEA) represents a numerical method, which provides solution to problems that would otherwise be difficult to obtain. It is an effective method of determining the static performance of structures for three reasons which are saving in design time, cost effective in construction and increase the safety of the structure. The analysis of a structure with ANSYS is performed in three stages:

a) Pre-processing P – defining the finite element model and environmental factors to be applied to it.

b) Analysis solver – solution of finite element model.

c) Post-processing of results like deformations contours for displacement, etc., using visualization tools.

2.2 Element Types used in Finite Element Model

(a)Reinforced concrete - The solid 65 element was used to model the concrete. This element has 8 nodes with 3degree of freedom at each node – translations in the nodal x, y, and z-directions. This element is capable of plastic deformation, cracking in three orthogonal direction, and crushing.

(b)Steel reinforcement – To model concrete reinforcing, one of two methods is following followed. In the first method, the reinforcing is simulated as spar element with geometric properties similar to the original reinforcing. These elements can directly be generated from the nodes in the model. This method of discretization is useful in simple concrete models. The second method of steel reinforcing is the smeared concrete element method. Cracks can also be analysed into either the discrete type or smeared type. Link 180 element is used to model steel reinforcement in this paper. This element is a 3D spar element. It has three nodes with two degrees of freedom – translations in the nodal x, y, and z-directions being also capable of plastic deformation.

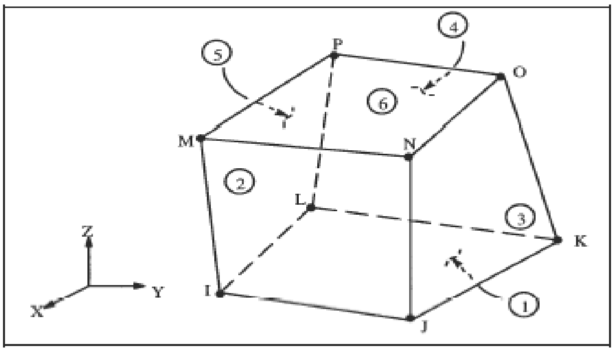

Figure 1. Solid 65 - 3D reinforced concrete solid element

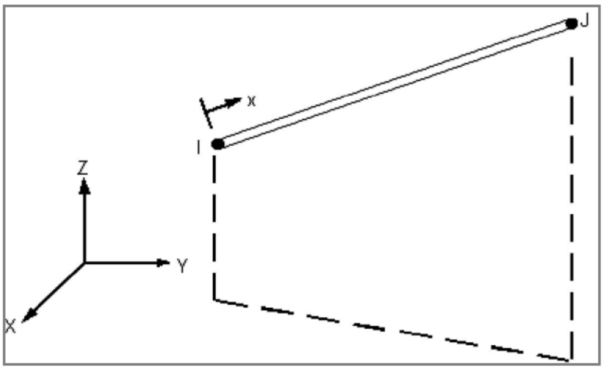

Figure 2. Link 180 element

2.3 Real Constants

Individual element contains different real constants. Real constant set 1 is used to for the solid 65element. Real constant sets 2, 3, 4, 5 are defined for the Link 180 element. Values of cross sectional area and initial strain are entered. A value of zero is entered for the initial strain because there is no initial strain in the reinforcement.

2.4 Modelling of the Beam – The beam can be modelled using solid 65 and link 180.

(a) Material Properties – Two material models are given: material 1 for concrete and material 2 for steel. Solid 65 element requires linear and multi-linear isotropic material properties to properly model concrete.

For material model 1(Group A)

Linear isotropic properties

Modulus of elasticity EX = 22360.67 N/mm^2

Poison's ratio PRXY = 0 .2

Multi-linear isotropic properties

In the present numerical analysis, the uniaxial behaviour of concrete was modelled by the numerical expression proposed by Desayi and Krishnan [1965] incorporating the modification proposed by Gere and Timoshenko [1997].

Points	M20	
	Strain	Stress
1.	0.000183	4.02
2.	0.000275	6.01
3.	0.0005	10.37
4.	0.0010	17.04
5.	0.0012	18.51
6.	0.0014	19.42
7.	0.0016	19.89
8.	0.0020	20
9.	0.0035	20

Open shear transfer co-efficient = 0.3

Closed shear transfer co-efficient = 1

Uniaxial cracking strength = 3.13 N/mm^2

Uniaxial crushing strength = 20 N/mm^2

For material model 2

Linear isotropic properties

Modulus of elasticity EX – 200000 N/mm^2

Poison's ratio PRXY – 0.3

Bilinear isotropic

Yield stress – 415 N/mm^2

Tangent modulus- 20 N/mm^2

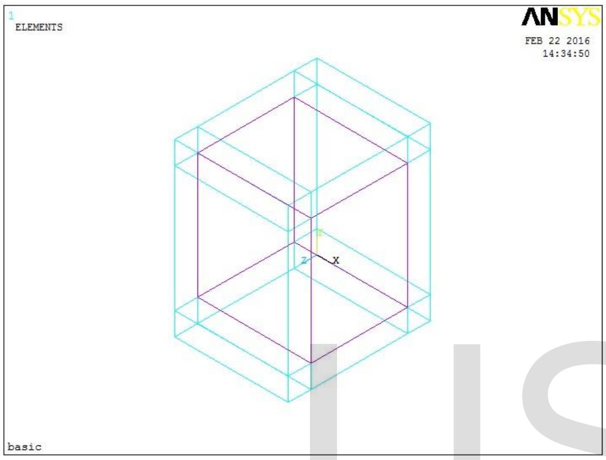

Figure 3. Element connectivity in FE model

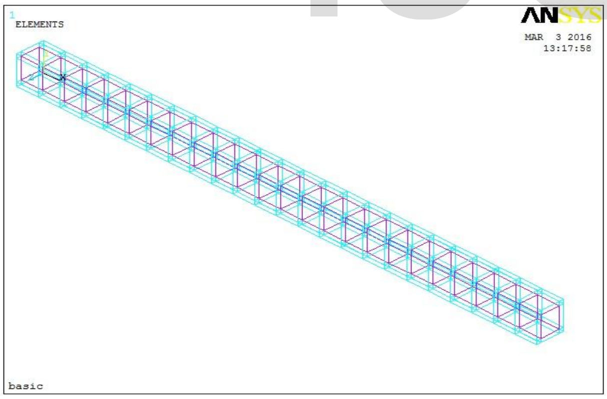

Figure 4. FE model of beam specimen

2.5 Loading and Boundary Condition

Horizontal and vertical restraints, representing a simply supported at both end of the beam. The load is applied at the center of the span.

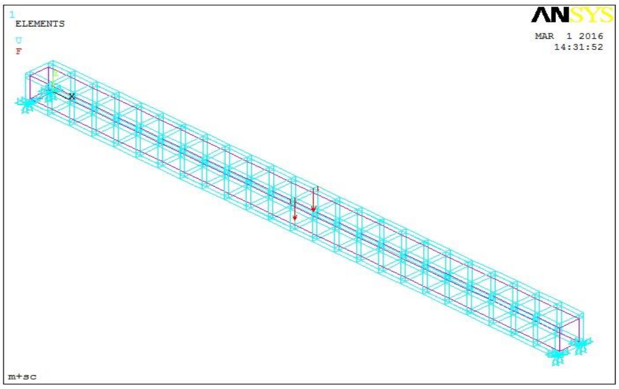

Figure 5. Complete FE Model with Loading and Boundary Conditions

2.6 Finite Element Discretization

The model is divided into a no. of small elements, and after loading stress and strain is calculated at integration points of these small elements.

2.7 Nonlinear Solution

In nonlinear solution, the total load applied to a finite element model is divided into a series of load increment called load steps. The ANSYS program uses Newton-Raphson equilibrium iterations for updating the model stiffness. In this study, for the reinforced concrete solid elements, convergence criteria is based on force and displacement, and the convergence tolerance limits is initially selected by the ANSYS program. It is found that convergence of solutions for the models is difficult to achieve due to the nonlinear behaviour of reinforced concrete. Therefore, the convergence tolerance limits is increased to a maximum of 5 times the default tolerance limits(0.5 % for force checking and 5% for displacement checking) in order to obtain convergence of the solutions.

2.8 Load Stepping and Failure Definition for FE Models

For the nonlinear analysis, automatic time stepping in the ANSYS program predicts and controls load step sizes. Based on the previous solution history and the physics of the models, if the convergence behaviour is smooth, automatic time stepping will increase the load increment up to a selected maximum load step size. If the convergence behaviour is abrupt, automatic time stepping bisect the load increment until it is equal to a selected minimum load step size. The maximum and minimum load step sizes are required for the automatic time stepping.

3. Finite Element Analysis

For the analysis of the RC beam, six combinations of different diameter of bars are used in tension reinforcement**.** The six models of M20, M25 and M30 grade of concrete with percentage of reinforcement varying from 0.54 to 1.26% are presented. IS : 456-2000 recommends minimum area of steel as (85% of effective area of cross section)/f_y and maximum area of steel as 4% of gross area of cross-section. Based on these recommendations, area of steel for various grades of concrete and steel used in analytical programme are given in tables 1, 2 and 3.

Table 1. combination of reinforcement for flexure test of Group A

Model no.	Diameter of bars(mm)	No. of bars	Area of Tension steel (sq.mm)	pt.(%)
A_1	2#16 ϕ	2	402.12	0.54
A_2	2#16 ϕ + 1#12 ϕ	3	515.22	0.69
A_3	3#16 ϕ	3	603.18	0.80
A_4	2#20 ϕ	2	628.32	0.84
A_5	2#20 ϕ + 1#16 ϕ	3	829.38	1.11
A_6	3#20 ϕ	3	942.48	1.26

Table 2. combination of reinforcement for flexure test of Group B

Model no.	Diameter of bars(mm)	No. of bars	Area of Tension steel (sq.mm)	pt.(%)
B_1	2#16 ϕ	2	402.12	0.54
B_2	2#16 ϕ + 1#12 ϕ	3	515.22	0.69
B_3	3#16 ϕ	3	603.18	0.80
B_4	2#20 ϕ	2	628.32	0.84
B_5	2#20 ϕ + 1#16 ϕ	3	829.38	1.11
B_6	3#20 ϕ	3	942.48	1.26

Table 3. combination of reinforcement for flexure test of Group C

Model no.	Diameter of bars(mm)	No. of bars	Area of Tension steel (sq.mm)	pt.(%)
C_1	2#16 ϕ	2	402.12	0.54
C_2	2#16 ϕ + 1#12 ϕ	3	515.22	0.69
C_3	3#16 ϕ	3	603.18	0.80
C_4	2#20 ϕ	2	628.32	0.84
C_5	2#20 ϕ + 1#16 ϕ	3	829.38	1.11
C_6	3#20 ϕ	3	942.48	1.26

4. Analysis of the Models

In the nonlinear region of the response, subsequent cracking occurs as more loads are applied to the beam. Cracking increases in the constant moment region, and the beam starts cracking outwards the support. Figures 6 to 17 represent the first crack and deflection for RCC beam specimen M20 grade of concrete.

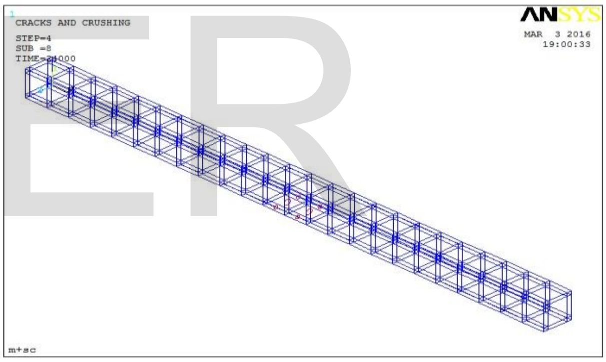

Figure 6. First Crack in model A_1

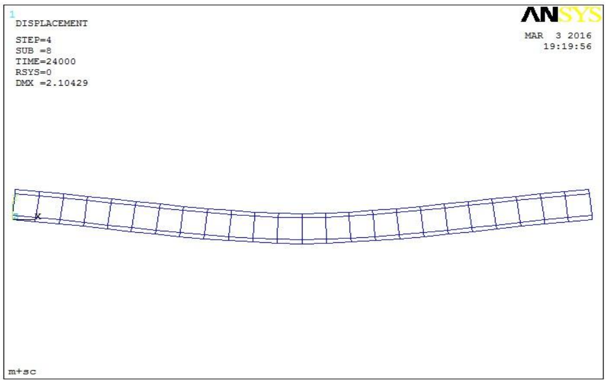

Figure 7. Deflection at First Crack in model A_1

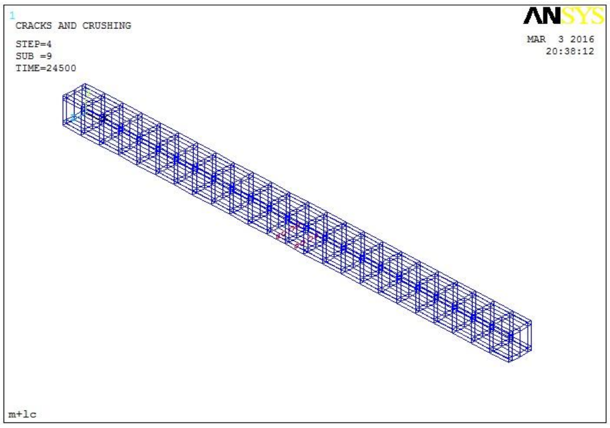

Figure 8. First Crack in model A_2

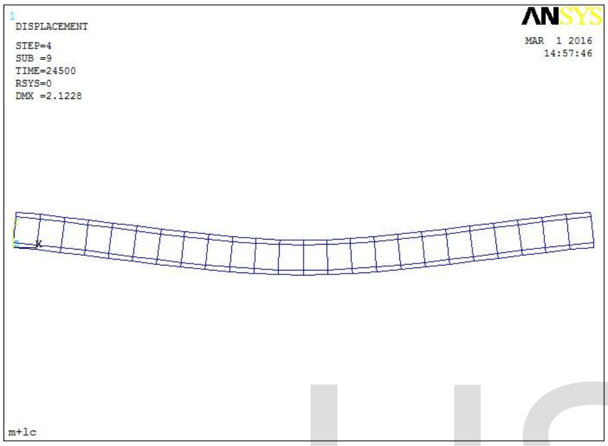

Figure 9. Deflection at First Crack in model A_2

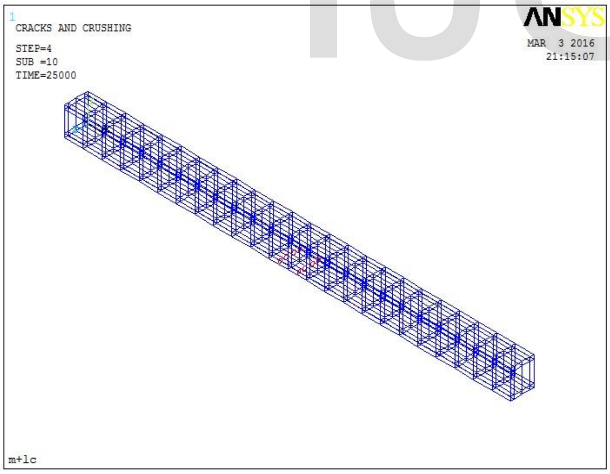

Figure 10. First Crack in model A_3

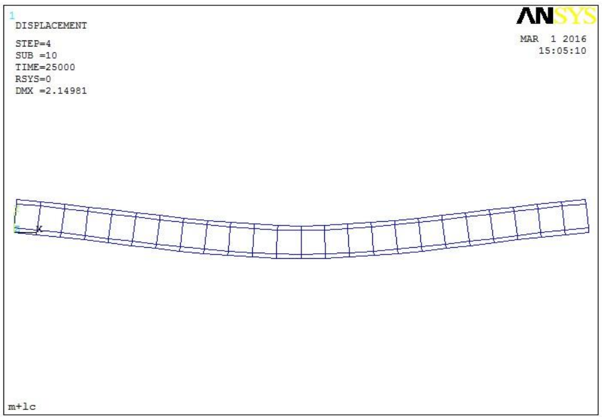

Figure 11. Deflection at First Crack in model A_3

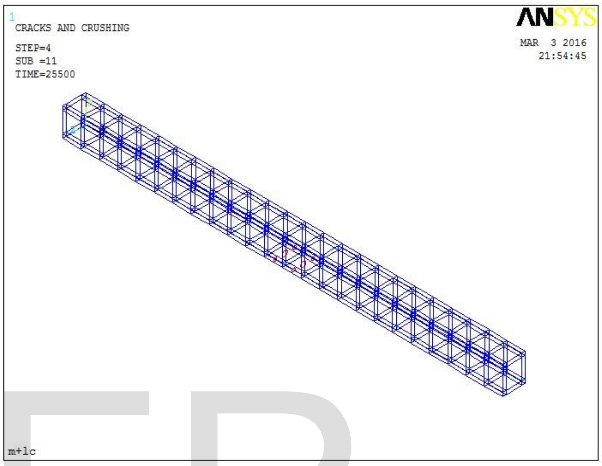

Figure 12. First Crack in model A_4

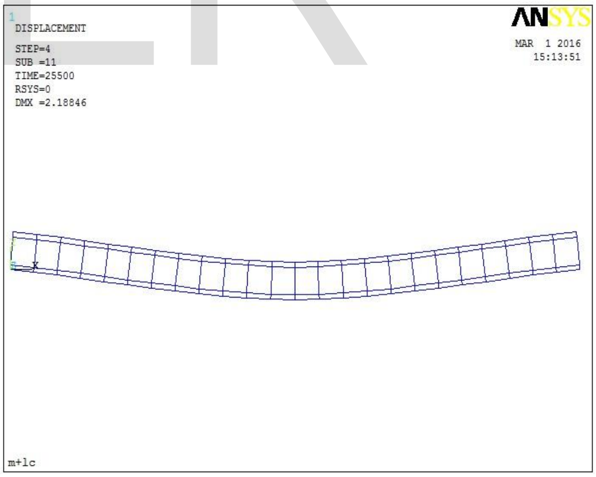

Figure 13. Deflection at First Crack in model A_4

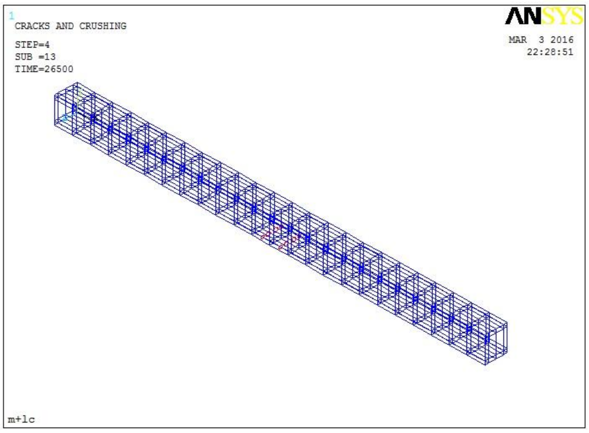

Figure 14. First Crack in model A_5

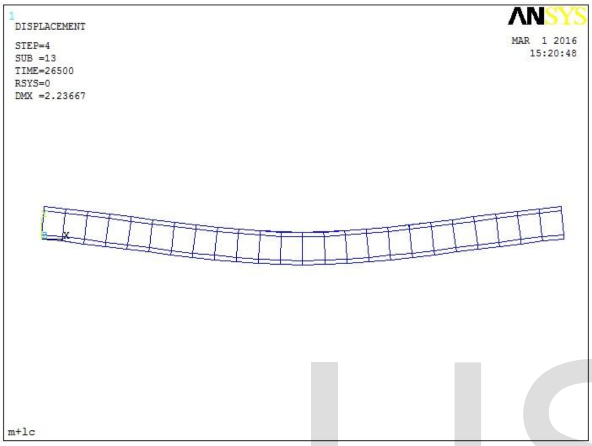

Figure 15. Deflection at First Crack in model A_5

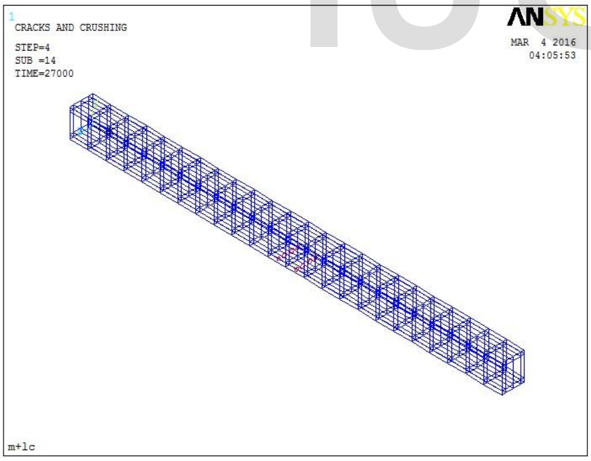

Figure 16. First Crack in model A_6

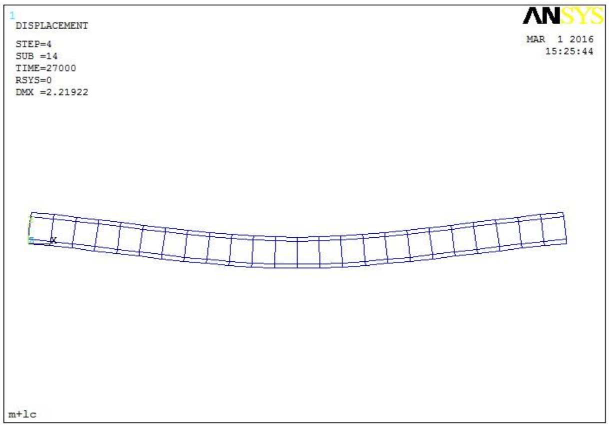

Figure 17. Deflection at First Crack in model A_6

Figures 18 to 23 represent load deflection curve for RCC beam specimen M20 grade of concrete. In these specimens, because of reinforcement, the load at elastic limit as well as the failure load was observed higher. The deflections measured for all the specimens were mid-span deflections.

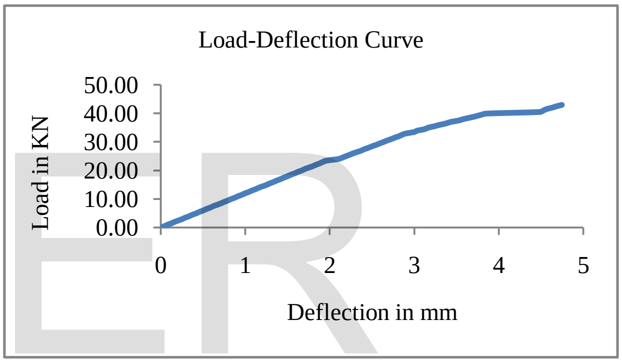

Figure 18. Load deflection curve for model A_1

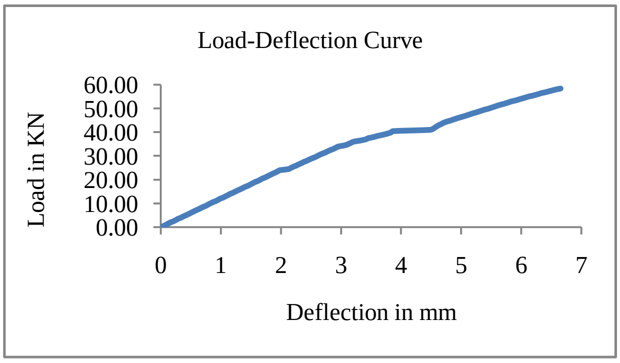

Figure 19. Load deflection curve for model A_2

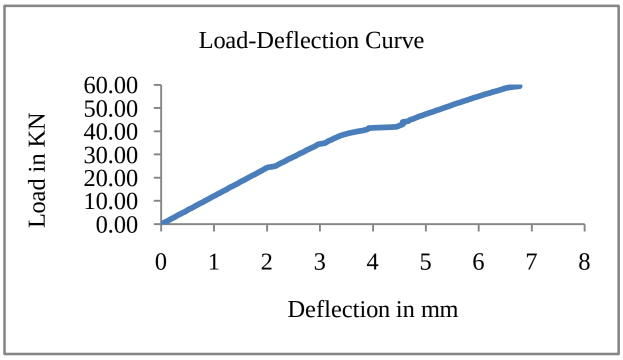

Figure 20. Load deflection curve for model A_3

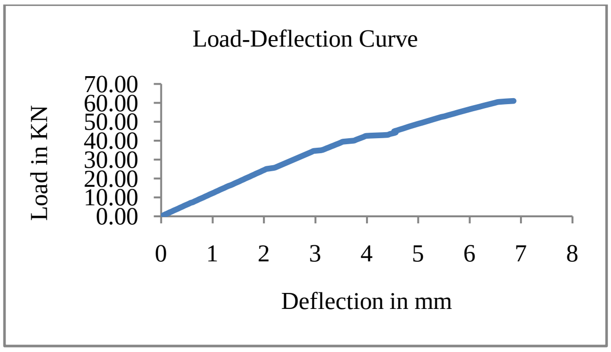

Figure 21. Load deflection curve for model A_4

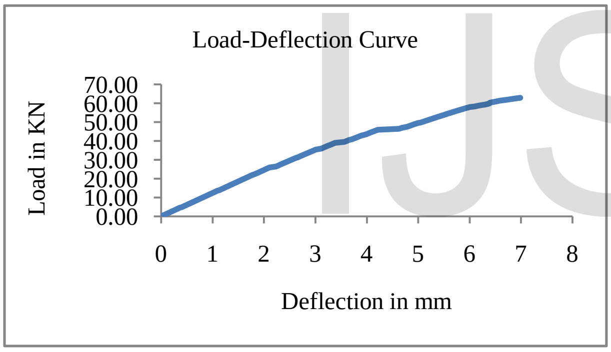

Figure 22. Load deflection curve for model A_5

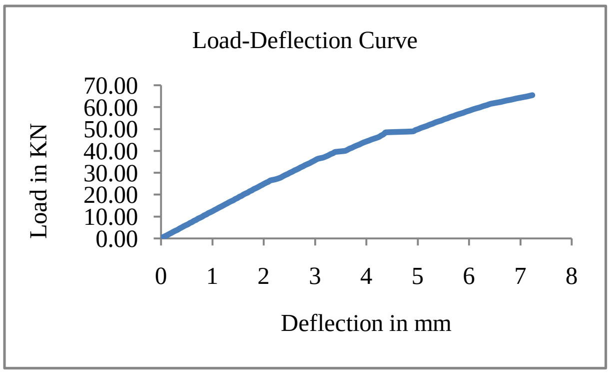

Figure 23. Load deflection curve for model A_6

4.1 Evaluation of 'E' of RC Beam for M20 Grade of Concrete

For a beam subjected to mid-point loading the deflection formula is as follows

$$\delta = \frac{5wl^4}{384EI} + \frac{Wl^3}{48EI}$$

where *w* is self-weight of beam in N/mm and W is the load at failure in N.

The value of mid-span deflection is substituted in the above equation for each case. By substituting proper values of w, W, *l*, and I, the remaining unknown value i.e., E is evaluated. The above eq. is applicable for linear behaviour only. The value of W substituted in this expression is corresponding to first crack. Graphical representation of the values indicated in table 4 is shown in figure 24.

Table 4. Percentage of steel and corresponding 'E' value of reinforced concrete for RCC (M20)

Model no.	Percentage of steel reinforcement	Modulus of elasticity(N/mm^2)
A_1	0.54	57665.06
A_2	0.69	58127.00
A_3	0.80	58349.34
A_4	0.84	58254.66
A_5	1.11	58830.31
A_6	1.26	60215.75

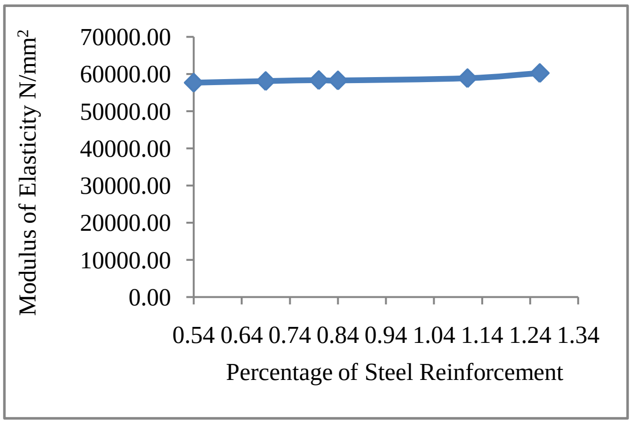

Figure 24. Modulus of elasticity Vs Percentage of reinforcement for M20

4.2 Evaluation of 'E' of RC Beam for M25 Grade of Concrete

Figures 25 to 30 are load-deflection curves for M25 grade for all specimens in table 2. The curve for each specimen is different because of reinforcement. The load carrying capacity of the specimens has increased due to increase in the percentage of reinforcement.

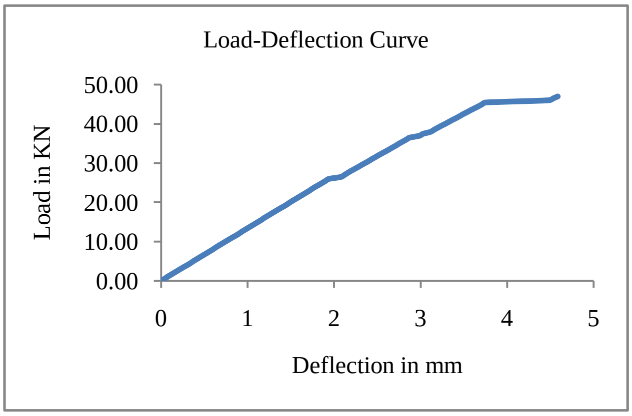

Figure 25. Load deflection curve for model B_1

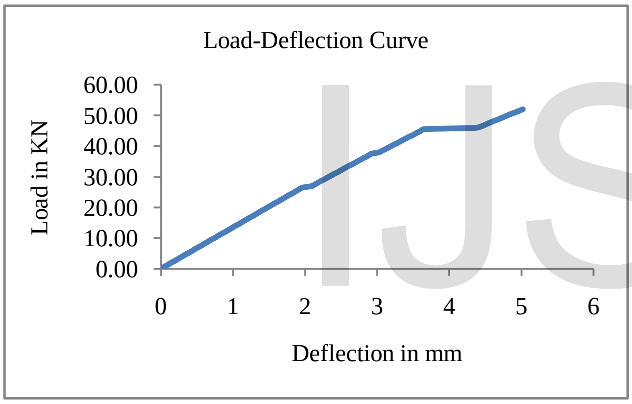

Figure 26. Load deflection curve for model B_2

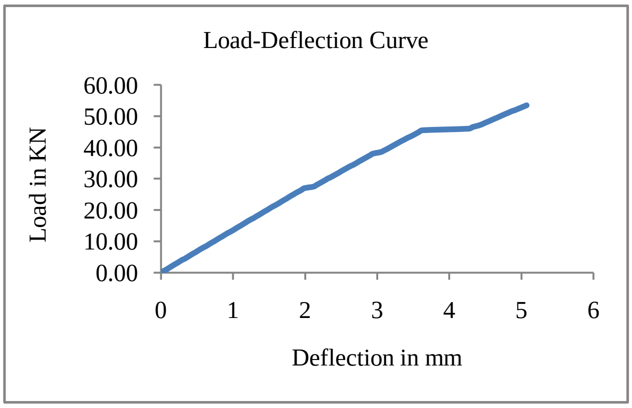

Figure 27. Load deflection curve for model B_3

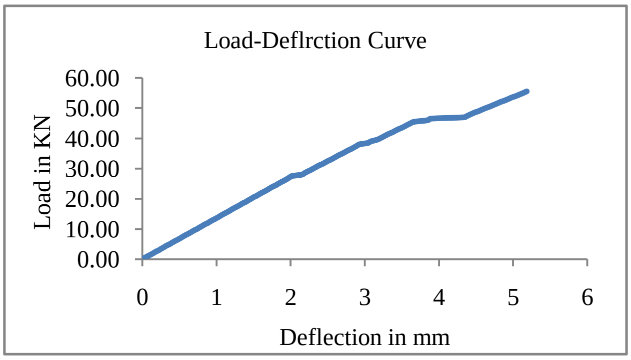

Figure 28. Load deflection curve for model B_4

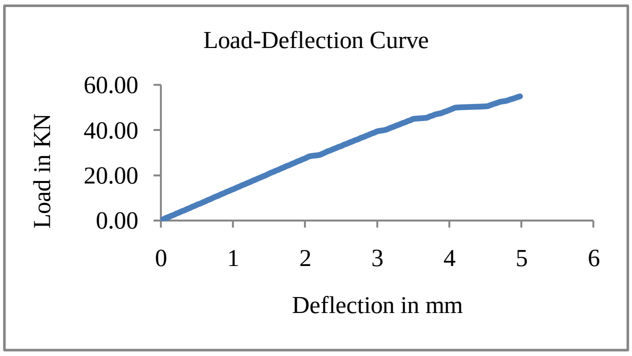

Figure 29. Load deflection curve for model B_5

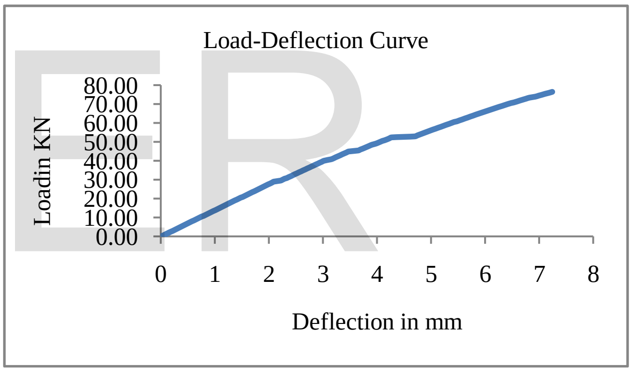

Figure 30. Load deflection curve for model B_6

E of RCC for M25 grade of concrete is also evaluated as per the procedure mentioned above

Table 5. Percentage of steel and corresponding 'E' value of reinforced concrete for RCC (M20)

Model no.	Percentage of steel reinforcement	Modulus of elasticity(N/mm^2)
B_1	0.54	63274.92
B_2	0.69	63756.03
B_3	0.80	63980.69
B_4	0.84	63908.83
B_5	1.11	64533.21
B_6	1.26	64886.42

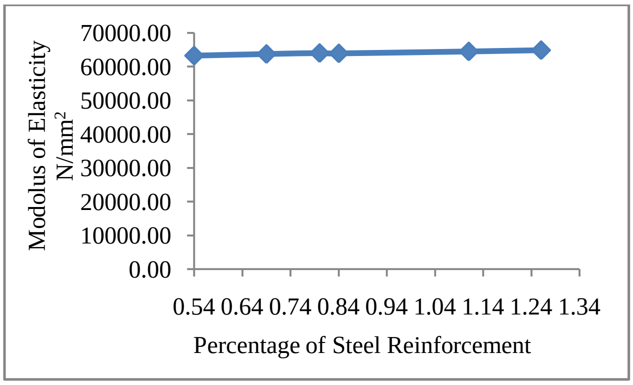

Figure 31. Modulus of elasticity Vs Percentage of reinforcement for M25

4.3 Evaluation of 'E' of RC Beam for M30 Grade of Concrete

Figures 32 to 37 are load-deflection curves for M30 grade for all specimens in table 3. The curve for each specimen is different because of reinforcement. The same trend was observed for specimens of M30 grade of concrete.

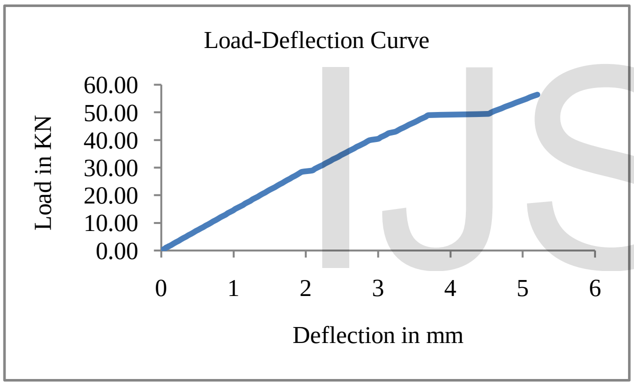

Figure 32. Load deflection curve for model C_1

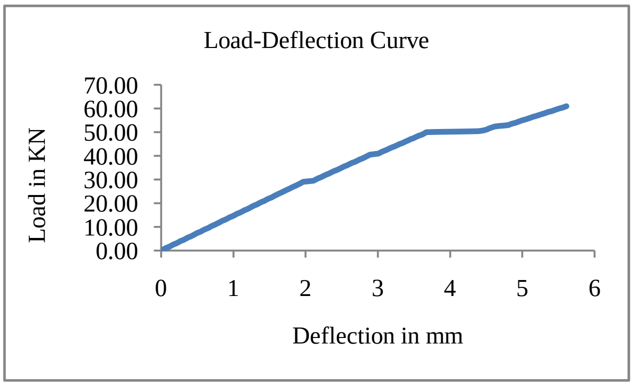

Figure 33. Load deflection curve for model C_2

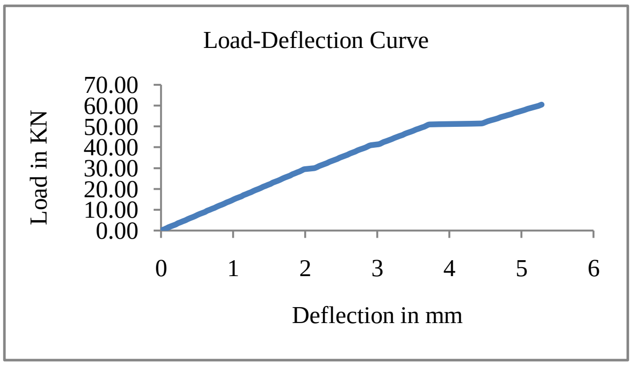

Figure 34. Load deflection curve for model C_3

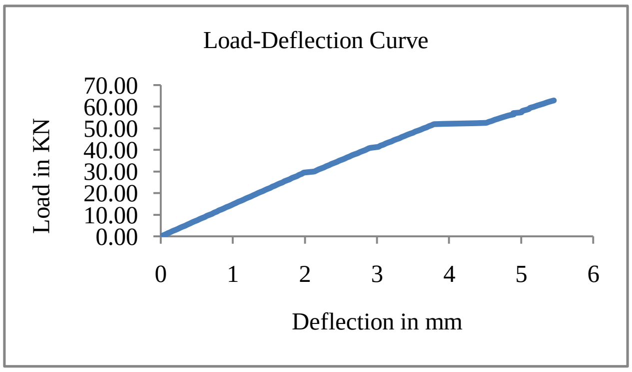

Figure 35. Load deflection curve for model C_4

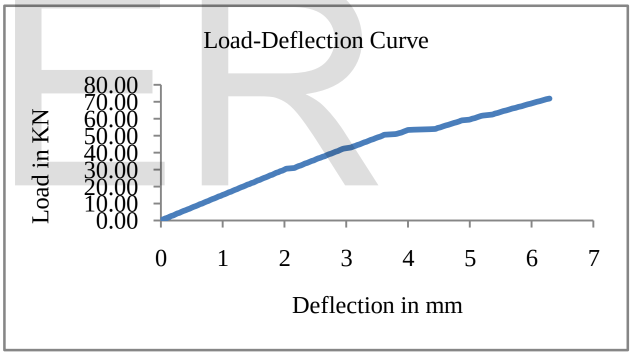

Figure 36. Load deflection curve for model C_5

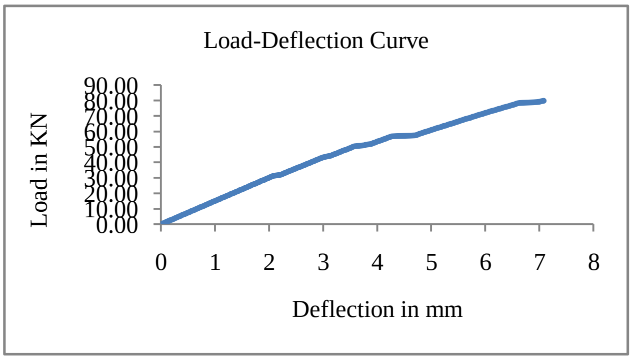

Figure 37. Load deflection curve for model C_6

Table 6.Percentage of steel and corresponding 'E' value of RCC (M30)

Model no.	Percentage of steel reinforcement	Modulus of elasticity(N/mm^2)
C_1	0.54	67931.51
C_2	0.69	68385.43
C_3	0.80	68682.78
C_4	0.84	68793.84
C_5	1.11	69474.37
C_6	1.26	72751.98

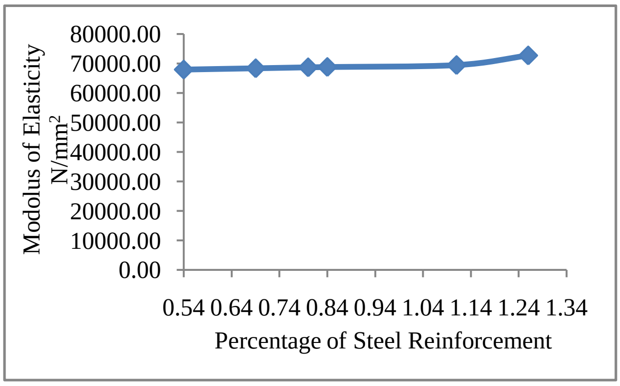

Figure 38. Modulus of elasticity Vs Percentage of reinforcement for M30

5. Conclusion

• IS: 456-2000 suggests the formula for modulus of elasticity of plain concrete (PCC) as

$$E = 5000\sqrt{f_{ck}}$$

which does not consider the effect of reinforcement. It was observed that modulus of elasticity of reinforced concrete varies with percentage of reinforcement and grade of concrete.

• Proposed values of 'E' for RCC Beam provides sufficient estimate prior to analysis of RC structures. It has been observed from the above result that the Modulus of Elasticity for RCC is more than the Modulus of Elasticity of PCC and hence deformations are expected to be on lower side.

• The load carrying capacity of the specimens was highly enhanced due to higher grade of concrete and reinforcement. When the beams reached the limit state, a sufficient time was available before failure.

• The range of tension reinforcement is 0.54 to 1.26% as per Indian Standard norms for RC beams. This specific range commensurate with geometric parameters of the model analysed.

6. Scope for Future Work

Additional work may be undertaken in the determination of modulus of elasticity:

(i) The results of three more grades of concrete namely M35, M40 and M45 can be obtained analytically. These results are used to obtain a generalized equation for modulus of elasticity of RC beam.

(ii) Effect of change in percentage in compression reinforcement with the change in tension reinforcement can be studied.

(iii) Variation in spacing of stirrups and depth of beam can also be studied.

(iv) The boundary conditions of the beam can be varied.

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